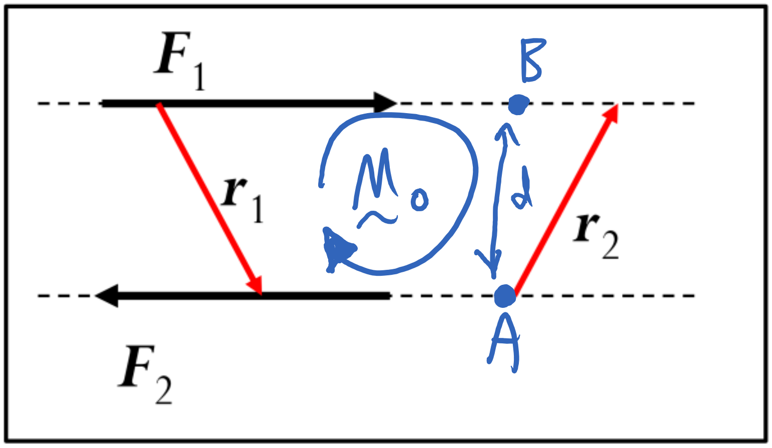
iClicker redo

Monday, February 6, 2017 10:37 AM



F_1 and F_2 form a couple. The moment of the couple is given by:

(a) $r_1 \times F_1$ (b) $r_2 \times F_1$
(c) $F_2 \times r_1$ (d) $r_2 \times F_2$

C.W.

gives C.C.W. rotation

rewrite as a couple ⇒ Note C.C.W. sense of action

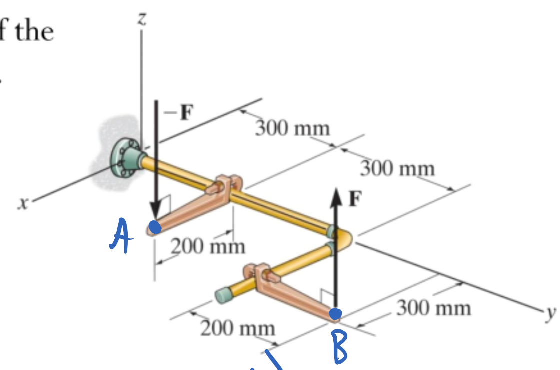
$\underset{\sim}{M}_o = \underset{\sim}{r}_2 \times \underset{\sim}{F}_1$ gives C.W. action

$\underset{\sim}{F}_2$ acts on the line through A.

∴ $\underset{\sim}{F}_2$ creates no moment about A.

Similarly, $\underset{\sim}{F_1}$ creates no moment about B.

Determine the magnitude and coordinate direction angles of the couple moment. The pipe line assembly lies in the x-y plane. Assume F = 80 N.



$$\underset{\sim}{r}_{AB} = \underset{\sim}{r}_B - \underset{\sim}{r}_A$$

$$\underset{\sim}{r}_B = (300\hat{i} + 800\hat{j})\ \text{mm}$$

$$\underset{\sim}{r}_A = (200\hat{i} + 300\hat{j})\ \text{mm}$$

$$\underset{\sim}{r}_{AB} = (100\hat{i} + 500\hat{j})\ \text{mm}$$

$$\underset{\sim}{F}_A = -80\hat{k}\ \text{N}$$

$$\underset{\sim}{F}_B = 80\hat{k}\ \text{N}$$

Couple moment:

$$\underset{\sim}{M}_O = \underset{\sim}{r}_A \times \underset{\sim}{F}_A + \underset{\sim}{r}_B \times \underset{\sim}{F}_B$$

$$= \underset{\sim}{r}_A \times (-\underset{\sim}{F}_B) + \underset{\sim}{r}_B \times \underset{\sim}{F}_B$$

$$= -\underset{\sim}{r}_A \times \underset{\sim}{F}_B + \underset{\sim}{r}_B \times \underset{\sim}{F}_B$$

$$= (-\underset{\sim}{r}_A + \underset{\sim}{r}_B) \times \underset{\sim}{F}_B$$

$$= (\underset{\sim}{r}_B - \underset{\sim}{r}_A) \times \underset{\sim}{F}_B$$

$$= \underset{\sim}{r}_{AB} \times \underset{\sim}{F}_B$$

$$= (100\hat{i} + 500\hat{j}) \times (80\hat{k})\ \text{N}\cdot\text{mm}$$

$$= (8000\,\underbrace{\hat{i}\times\hat{k}}_{-\hat{j}} + 40000\,\underbrace{\hat{j}\times\hat{k}}_{\hat{i}})\ \text{N}\cdot\text{mm}$$

$$= (40\hat{i} - 8\hat{j})\ \text{N}\cdot\text{m}$$

Magnitude:

$$M_o = |\underset{\sim}{M}_o| = \sqrt{40^2 + (-8)^2}\ N\cdot m$$

$$= \sqrt{1664}\ N\cdot m$$

$$\simeq 40.8\ N\cdot m$$

Direction Cosines:

Unit vector

$$\hat{u}_m = \frac{40\hat{i} - 8\hat{j}}{\sqrt{1664}} \simeq \underbrace{\frac{40}{40.79}}_{\cos\alpha}\hat{i} - \underbrace{\frac{8}{40.79}}_{\cos\beta}\hat{j}$$

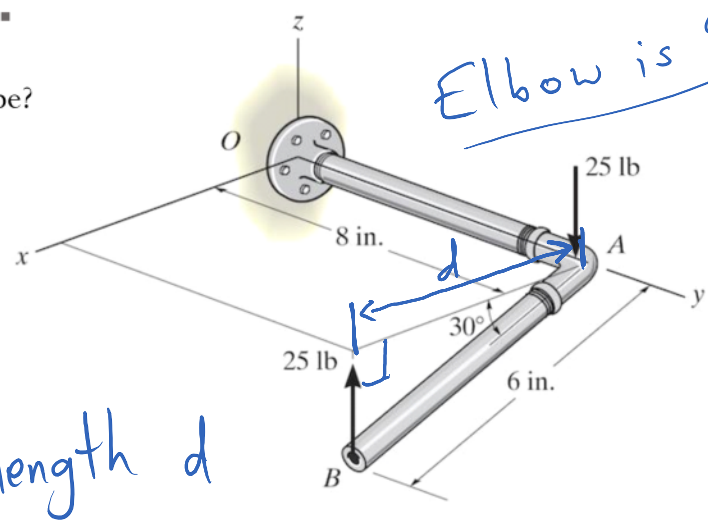
$$\therefore\ \alpha = \cos^{-1}\left(\frac{40}{40.79}\right) = 11.3^\circ$$

$$\beta = \cos^{-1}\left(\frac{-8}{40.79}\right) = 78.7^\circ$$

I>Clicker question:

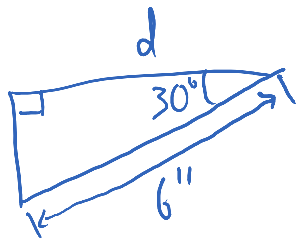
4) What is the couple moment acting on the pipe?

A. $+150\mathbf{j}$ lb·in
B. $+130\mathbf{j}$ lb·in
C. $+75\mathbf{j}$ lb·in
D. $-75\mathbf{j}$ lb·in
E. $-130\mathbf{j}$ lb·in



Elbow is 90°

Find moment arm length d ⊥ to $\underset{\sim}{F}$.



$d = 6'' \cdot \cos 30°$
$= 6'' \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$ in.
$\approx 5.196''$

$M = F \cdot d$ in $-\hat{j}$ direction
$= (25 \text{ lbs})(5.196'') = 130$ lb-in

$\underset{\sim}{M} = -130\,\hat{j}$ lb·in.

Page 16

Tuesday, January 31, 2017 10:53 PM

Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

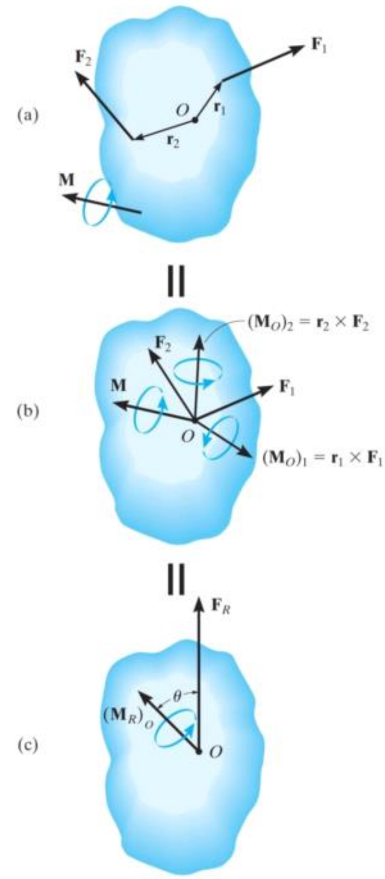
Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point *P*.

Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_O$:

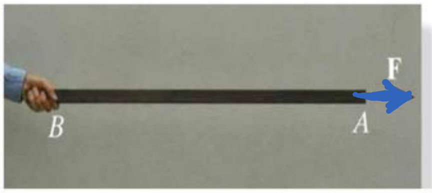
$$\boldsymbol{F}_R = \sum \boldsymbol{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2$$

in diagrams on the right

$$(\boldsymbol{M}_R)_O = \underbrace{\sum \boldsymbol{M}_O}_{\text{moments caused by } \underset{\sim}{F}_1 \,\&\, \underset{\sim}{F}_2 \text{ about O}} + \underbrace{\sum \boldsymbol{M}}_{\text{applied couples}}$$



Moving a force on its line of action



Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

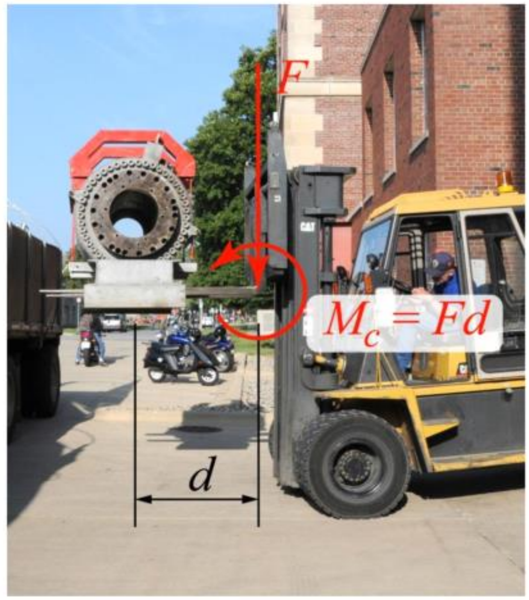
However, the **internal effect** of the force on the body does depend on where the force is applied.

Moving a force off of its line of action

Tuesday, January 31, 2017 10:53 PM

Moving a force off of its line of action

Force system I



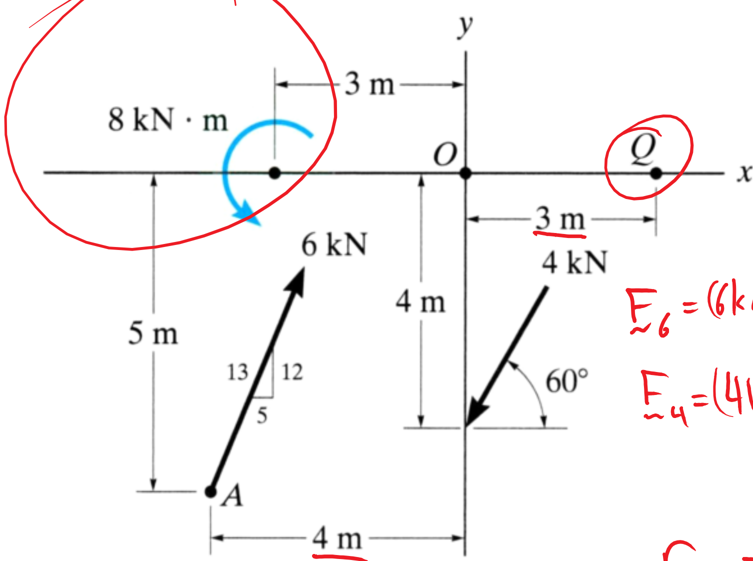
Force system II

The two force systems are equipollent since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.

So moving a force off its line of action means you have to "add" a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.

Problem

Replace the force and couple system by an equipollent force and couple moment at point Q.



$\underset{\sim}{F}_6 = (6kN)\left(\frac{5}{13}\hat{i} + \frac{12}{13}\hat{j}\right)$

$\underset{\sim}{F}_4 = (4kN)\left(\underbrace{-\cos 60^\circ}_{-1/2}\hat{i} - \underbrace{\sin 60^\circ}_{-\frac{\sqrt{3}}{2}}\hat{j}\right)$

$\underset{\sim}{r}_{Q6} = (-7\hat{i} - 5\hat{j})\,m$

$\underset{\sim}{r}_{Q4} = (-3\hat{i} - 4\hat{j})\,m$

Moments about Q:

$M_{Q6} = \underset{\sim}{r}_{Q6} \times \underset{\sim}{F}_6 = (-7\hat{i} - 5\hat{j}) \times \left(\frac{30}{13}\hat{i} + \frac{72}{13}\hat{j}\right) kN \cdot m$

$= -27.23\hat{k} \cdot kN \cdot m$

$M_{Q4} = \underset{\sim}{r}_{Q4} \times \underset{\sim}{F}_4 = (-3\hat{i} - 4\hat{j}) \times (-2\hat{i} - 2\sqrt{3}\hat{j})\, kN \cdot m$

$= 2.392\hat{k}\, kN \cdot m$

Find sum of all moments & couples

$$\sum M = (8\,kN\cdot m\,\hat{k}) - (27.23\cdot kN\cdot m\cdot\hat{k}) + (2.392\cdot kN\cdot m\cdot\hat{k})$$

$$= -16.84\cdot\hat{k}\cdot kN\cdot m$$

Resultant Force:

$$\underset{\sim}{F}_6 + \underset{\sim}{F}_4 = (0.308\,\hat{i} + 2.07\,\hat{j})\,kN$$